'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

GE1-P2-STATISTICS

FUNDAMENTAL OF PROBABILITY THEORY

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions — $1\times5=5$

(a) A speak truth in 75% and B in 80% of the cases. In what percentages of cases are they likely to contradict each other in stating the same fact?

(b) Give chief features of the normal curve.

(c) The probability density function of a random variable X is

$$f(x)=\frac{1}{\theta}e^{-\frac{x}{\theta}},\ x>0,\ \theta>0$$
$$=0,\quad \text{otherwise}$$

Find $E(X^2)$.

(d) If events A and B are not mutually exclusive, then show that

$$P(AB)\geq P(A)+P(B)-1.$$

(e) State the central limit theorem.

(f) Explain discrete probability distribution.

(g) For a binomial distribution with mean 5 and S.D. 2, find the mode.

(h) State two properties of hypergeometric distribution.

GROUP-B

2. Answer any ***three*** questions: — $5\times3=15$

(a) State and prove Bayes' theorem.

(b) If X follows binomial distribution with parameter n and p then prove that $P[X \text{ is even}]=\frac{1}{2}[1+(q-p)^n]$, where $p+q=1$.

(c) Show that the expectation of the sum of two jointly distributed random variables X and Y is the sum of their expectations.

(d) The joint p.d.f of (X, Y) is given by

$$f(x, y) = 2 ; \quad 0 < x < 1, \quad 0 < y < x$$
$$= 0 \quad ; \quad \text{otherwise}$$

Find the marginal density of X and the conditional density of Y. (Given $X = x$).

(e) If X is a Poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$
Find the mean of X.

GROUP-C

3. Answer any ***two*** questions: $10\times 2 = 20$

(a) Show that the mean and variance of the normal distribution are μ and σ^2 respectively.

(b) (i) Let X be a binomially distributed random variable with parameters n and p. For what value of p is var (X) a maximum, if you assume that n is fixed?

(ii) Derive Poisson distribution as the limit of binomial distribution.

(c) (i) Two unbiased dice are thrown. Find the expected value of the sum of the numbers of points on them.

(ii) Find the points of inflection of the normal curve.

(d) (i) Find the mode of the binomial distribution.

(ii) Two persons toss a true coin n times each. Show that the probability of their scoring the same number of heads is $\binom{2n}{n}2^{-2n}$.

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